DELAWARE VALLEY SCHOOL DISTRICT

PLANNED INSTRUCTION

A PLANNED COURSE FOR:

ADVANCED TOPICS (1st SEMESTER)

Grade Level: 12

Date of Board Approval: __2014________________

DELAWARE VALLEY SCHOOL DISTRICT

PLANNED INSTRUCTION

Title of Planned Instruction: Advanced Topics (1st semester)

Subject Area: Mathematics Grade Level: 12

Course Description:

This course is designed for advanced mathematics students to be taken in conjunction with Advanced Placement Calculus (either AB or BC). The course focuses on foundations of mathematics, including logic, methods of mathematical proof, writing and critiquing of proofs, number theory, set theory (including theory of infinite sets), and paradoxes. The course will also emphasize development of students' mathematical problem solving skills in a variety of areas including algebra, geometry, probability, logic, and word problems in order to be successful in mathematics contests at the local, state, and national levels. Furthermore, the course will stress students' proper use of terminology and precise language both verbally and in writing.

Time/Credit for the Course: 46 minutes per day for 1 semester/ ½ credit

Curriculum Writing Committee: Gary Dennis

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Resources for Advanced Topics (1st semester)

Euclid in the Rainforest by Joseph Mazur

Foundations of Higher Mathematics by Peter Fletcher and C. Wayne Patty

Flatland by Edwin Abbott

Hexaflexagons and Other Mathematical Diversions by Martin Gardner

History of Mathematics by David M. Burton

Journey Through Genius – The Great Theorems of Mathematics by William Dunham

The Man Who Loved Only Numbers – The Story of Paul Erdös and the Search for Mathematical Truth by Paul Hoffman

Mathematical Fallacies, Flaws, and Flimflam by Edward J. Barbeau

The Mathematical Universe by William Dunham

“Paradox Lost” by Ian Stewart

Penrose Tiles to Trapdoor Ciphers by Martin Gardner

Proofs Without Words – by Roger B. Nelsen

“Welcome to the Dark Side” by Ron Rosenbaum

Logic puzzles from various issues of “Games” magazine

“Joy of Mathematics” video series

Websites: www.mathworld.com (various topics and sites)
http://illuminations.nctm.org/ActivityDetail.aspx?ID=40
http://www.digicc.com/fido/
http://www-history.mcs.st-and.ac.uk/history/HistTopics/Perfect_numbers.html
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html
www.numberphile.com
http://www.cut-the-knot.org/pythagoras/index.shtml
http://www.squiglysplayhouse.com/Games/Java/Bridges/Bridges.html

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Problems and solutions from previous mathematics contests such as the Pennsylvania Mathematics League, the American Mathematics Competition, and the American Invitational Mathematics Exam

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Curriculum Map

First Marking Period

Logic and Language of Proofs

- Propositions: conjunctions, disjunctions, negations, conditionals, and biconditionals
- Negate propositions
- Truth tables
- Converses, contrapositives, and inverses of conditionals
- Word problems involving principles of logic

Expressions and Tautologies

- Propositional expressions and logical equivalence
- Tautologies and contradictions
- Proofs of tautologies
- DeMorgan's Laws

Quantifiers

- Truth sets of propositional functions
- Existential and universal quantifiers
- Negate propositional functions
- Counterexamples to false propositions

Number Theory

- Basic terms of number theory
- Sieve of Eratosthenes
- Fermat's Last Theorem and its history
- Binary number system and arithmetic of binary numbers
- Tower of Hanoi
- Fibonacci Sequence
- Sequences of numbers
- Properties of specific numbers

Methods of Proof

- Direct, contrapositive, and contradiction methods of proof
- Proof of the irrationality of $\sqrt{2}$
- Element-Chasing method of proof
- Parity

Set Theory

- Basic terms of set theory

- Set theory notation
- Formation of valid statements involving sets

Second Marking Period

Operations on Sets

- Universal sets
- Unions, intersections, and complements of sets
- Venn Diagrams
- Proof of the equivalence of two sets
- Proof of DeMorgan's Laws

Families of Sets

- Families of sets and indexed families of sets
- Unions and intersections of families of sets
- Proofs of theorems involving families of sets

Mathematical Induction

- Inductive sets
- Principle of Mathematical Induction
- Proofs of theorems using the Principle of Mathematical Induction
- Proof of algebraic inequalities
- Extended Principle of Mathematical Induction

Flatland

- Reading and analysis of Flatland by Edwin Abbott

Proofs Without Words

- Basic proofs without words
- Interpretation and creation of diagrams for proofs without words
- Proofs without words of the Pythagorean Theorem

Infinite Sets

- Attempts to define limits of functions
- Formal definition of a limit of a function
- Equinumerous sets and one-to-one functions
- Georg Cantor's history of infinite set theory
- Countably infinite and uncountably infinite sets (cardinality of infinite sets)
- Algebraic and transcendental numbers
- Cardinality of power sets

Paradoxes

- Definition of a paradox
- Zeno's paradox, Protagoras' paradox, the Pop Quiz paradox, and Simpson's Paradox
- Russell's paradox and infinite set theory paradoxes

Unit One: Logic and Language of Proofs

Big Idea #1: The laws of logic govern the structure and presentation of mathematical proofs.

Essential Questions:
- What are the principles of logic?
- How can the principles of logic be applied to mathematical proofs?

Concepts:
- Propositions and their truth tables
- Complex propositions
- Negations of propositions
- Truth tables
- Conditional statements and their converses, inverses, and contrapositives
- Principles of logic applied to word problems

Competencies:
- Determine the truth table for a proposition
- Form complex propositions
- Analyze the converse, inverse, and contrapositive of a conditional statement
- Solve word problems and puzzles using logic
- Analyze complex syllogisms

Big Idea #2: Propositional expressions can be formed to represent mathematical statements.

Essential Questions:
- How can propositional expressions and tautologies be analyzed?

Concepts:
- Propositional expressions
- Tautologies and contradictions
- DeMorgan’s Laws
- Proving tautologies

Competencies:
- Analyze propositional expressions
- Prove DeMorgan’s Laws
- Establish truth values of propositions using DeMorgan’s Laws
- Prove common tautologies

Big Idea #3: Quantifiers can be used to formulate propositions.

Essential Question:

- How can quantifiers be used to express mathematical statements?

Concepts:

- Existential quantifiers
- Universal quantifiers
- Negations of quantifiers
- Counterexamples of false propositions

Competencies:

- Explain the truth set of propositional functions
- Write propositional functions using quantifiers
- Interpret propositional functions
- Negate quantifiers
- Identify counterexamples to false propositions

Unit Two: Number Theory

Big Idea #1: Whole numbers possess a variety of properties, which can be analyzed to create general propositions.

Essential Questions:

- What properties do whole numbers possess?
- What general statements can be made about whole numbers?
- How can propositions about whole numbers be proven?
- How can properties of whole numbers be applied?

Concepts:

- Number theory terms
- Fermat's Last Theorem
- Binary numeral system
- Special numbers
- Numerical processes

Competencies:

- Understand and identify basic number theory terms
- Use the Sieve of Eratosthenes
- Formulate and justify number theory propositions
- Explain Fermat's Last Theorem
- Convert between decimal and binary numbers
- Perform arithmetic with binary numbers
- Apply the binary number system
- Solve the Tower of Hanoi problem and apply it to binary numbers
- Perform and analyze recursive processes on whole numbers
- Explain Graham's Number

Unit Three: Methods of Writing Proofs

Big Idea #1: Mathematical propositions can be verified through systematic methods of proof.

Essential Questions:
- How are mathematical statements formally proven?
- What is an appropriate method for justifying a statement?

Concepts:
- Direct Proofs
- Proofs by Contrapositive
- Proofs by Contradiction
- Flawed proofs
- Element-chasing method
- Parity

Competencies:
- Justify a mathematical statement using the Direct Method of Proof
- Justify a mathematical statement using the Contrapositive Method of Proof
- Justify a mathematical statement using the Contradiction Method of Proof
- Analyze and modify flawed proofs
- Use the concept of parity to solve problems
- Justify a mathematical statement using the Element-chasing Method of Proof
- Analyze the structures of polyhedra

Unit Four: Set Theory

Big Idea #1: The theory of sets is the fundamental system of mathematics on which all other mathematical theory is founded.

Essential Questions:
- How is a set defined?
- How are sets combined to create new sets?
- What general mathematical statements can be formulated about sets?

Concepts:
- Sets
- Universal sets
- Unions
- Intersections

- Complements

- Subsets
- Power sets
- Venn Diagrams
- Proofs
- DeMorgan’s Laws
- Families of sets
- Mathematical Induction
- Proofs without words
- Infinite sets
- Paradoxes

Competencies:

- Form the unions, intersection, complement, and power sets of given sets
- Use proper notation for sets, power sets, and the empty set
- Draw a Venn Diagram to represent unions, intersections, and complements
- Prove general statements involving sets
- Prove DeMorgan’s Laws for sets
- Form unions and intersections of families of sets
- Apply the Principle of Mathematical Induction to verify a proposition
- Interpret proofs without words
- Create proofs without words for a proposition
- Analyze infinite sets and their cardinality
- Analyze various paradoxes
- Explain Russell’s Paradox in terms of set theory

Unit Five: Flatland

Big Idea #1: Edwin Abbott's Flatland is a classic mathematics book that provides a perspective into higher dimensions.

Essential Questions:
- Why is the social structure in Flatland the way it is?
- How did the narrator of the book become aware of another dimension?
- What analogies and examples are given that describe higher dimensions?
- What implications does the book suggest to our three dimensional world?

Concepts:
- Geometry
- Polygons
- Perspective
- Social structure
- Higher dimensions
- Arithmetic and geometric progressions

Competencies:
- Explain the social structure of Flatland
- Analyze the issues that arise from the structure of Flatland
- Explain the difficulties of describing higher dimensions
- Describe the interactions that the narrator had with the inhabitants of Pointland and Lineland.

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Curriculum Plan

Unit 1: Logic and the Language of Proofs

Time Range in Days: 20 Days

Overview: Students will use the principles of logic and proper mathematical language and notation to analyze and formulate propositions.

Focus Questions:

- What are the principles of logic?
- How can the principles of logic be represented by truth tables?
- How can propositional expressions and tautologies be analyzed?
- How can quantifiers be used to express mathematical statements?
- How can the principles of logic be applied to mathematical proofs?

Goals: Students will be able to

- Recognize and follow the principles of logic
- Represent the principles of logic through truth tables
- Use quantifiers to express mathematical statements
- Apply the principles of logic to solve problems

Objectives: Students will be able to

- Recognize propositions and determine their truth values. (DOK Level Two)
- Form complex propositions using conjunctions, disjunctions, negations, conditionals, and biconditionals. (DOK Level Three)
- Know the definitions for conjunctions, disjunctions, negations, conditionals, and biconditionals both verbally and using truth tables. (DOK Level Two)
- Negate propositions. (DOK Level Two)
- Write a truth table for a given complex proposition. (DOK Level Three)
- Determine a correct proposition for a given truth table. (DOK Level Three)
- Analyze conditional statements and identify their hypothesis and conclusion. (DOK Level Three)
- Form the converse, contrapositive, and inverse of conditionals. (DOK Level Two)
- Express mathematical propositions both verbally and in writing using proper terminology and precise language. (DOK Level Two)
- Solve word problems that involve principles of logic. (DOK Level Four)
- Analyze complex syllogisms. (DOK Level Four)
- Analyze, verbalize, and write propositional expressions. (DOK Level Three)

- Identify, analyze, verbalize, and write logically equivalent expressions using principles of logic and truth tables. (DOK Level Three)
- Identify, analyze, verbalize, and write tautologies and contradictions using principles of logic and truth tables. (DOK Level Four)
- Prove DeMorgan's Laws using truth tables. (DOK Level Three)
- Explain both verbally and in writing the validity of DeMorgan's Laws. (DOK Level Four)
- Use DeMorgan's Laws to establish the truth values of other propositions. (DOK Level Three)
- Recognize and prove common tautologies, such as transitivity, distributivity, and associativity. (DOK Level Three)
- Determine and explain the truth set of a propositional function. (DOK Level Three)
- Recognize existential quantifiers, and write propositional functions using existential quantifiers. (DOK Level Two)
- Recognize universal quantifiers, and write propositional functions using universal quantifiers. (DOK Level Two)
- Use standard mathematical notation to write propositional functions. (DOK Level Three)
- Interpret propositional functions, and explain them both verbally and in writing using proper terminology and precise language. (DOK Level Three)
- Negate propositional functions involving existential and universal quantifiers. (DOK Level Three)
- Determine and explain both verbally and in writing counterexamples to false propositions. (DOK Level Three)

Core Activities:

- Direct instruction on the principles of logic.
- Guided practice with determining the truth table for complex propositions.
- Guided practice and independent student investigation of logic puzzles.
- Classroom discussion on the differences and logical equivalence between conditionals and their converse, inverse, and contrapositive.
- Guided practice and independent student investigation of syllogisms.
- Direct instruction on identifying and explaining tautologies and contradictions.
- Guided practice on the justification of DeMorgan's Laws using truth tables.
- Direct instruction on the differences and examples of existential and universal quantifiers.
- Direct instruction and guided practice on negating propositions involving quantifiers.

Assessments:

Diagnostic:

- Teacher observation and questioning
- Pre Calculus Final Exam

Formative:

- Teacher observation and questioning
- Homework assignments
- Graded homework assignment
- Group assignments
- Quizzes

Summative: Common assessment for Unit 1

Extensions:

- Extra problems from the textbook
- Additional logic puzzles from "Games" magazine
- Teacher designed problems

Correctives:

- Re-teaching
- Teacher designed problems

Materials and Resources:

- <u>Foundations of Higher Mathematics</u> by Peter Fletcher and C. Wayne Patty, p. 1-24
- Logic puzzles from "Games" Magazine

Unit 2: Number Theory

Time Range in Days: 10 Days

Overview: This unit explores the many properties and connections between the whole numbers, as well as the applications of theorems developed.

Focus Questions:

- What properties do whole numbers possess?
- What general statements can be made about whole numbers?
- What is the history of number theory?
- How can propositions about whole numbers be proven?
- How can properties of number theory be applied?

Goals: Students will be able to

- Identify and generalize properties of whole numbers
- Discuss the history of number theory
- Justify propositions in number theory
- Apply properties of number theory

Objectives: Students will be able to

- Define basic number theory terms: multiple, divisible by, factor/divisor, proper factor, prime, twin prime, Ruth-Aaron pairs, perfect square, perfect number, deficient number, superabundant number, odd, and even. (DOK Level One)
- Provide examples of each of the terms above. (DOK Level Two)
- Use the Sieve of Eratosthenes to identify prime numbers less than 100, and explain its usefulness in determining other prime numbers. (DOK Level Two)
- Formulate valid statements using the terms above, and express them using proper terminology and precise language. (DOK Level Three)
- Explain Fermat's Last Theorem and its history, and identify mathematicians who have attempted to solve it. (DOK Level Three)
- Know the definitions of the following sets of numbers and their standard mathematical symbols: natural numbers, integers, rational numbers, irrational numbers, real numbers, and complex numbers. (DOK Level One)
- Write decimal numbers in binary form. (DOK Level Two)
- Write binary numbers in decimal form. (DOK Level Two)
- Compute sums, differences, products, and quotients of numbers written in binary form. (DOK Level Three)

- Identify properties of numbers written in binary form, including odd/even, divisibility, and shortcuts to converting into decimal form. (DOK Level Three)
- Identify and explain the applications of binary code to computers and logic. (DOK Level Three)
- Identify and explain connection between the Icosian Game and the Tower of Hanoi, and explain their connections to binary numbers. (DOK Level Four)
- Describe the contributions of Charles Peirce to logic and computers, and recognize that he lived in Milford, PA. (DOK Level One)
- Explain the recursive process that leads to Kaprekar's constant. (DOK Level Three)
- Analyze how the Fido Puzzle works. (DOK Level Three)
- Identify Fibonacci Numbers and explain their applications. (DOK Level Three)
- Explain how Graham's Number is expressed and from where it is derived. (DOK Level Three)

Core Activities:

- Direct instruction on Charles S. Peirce and his contributions to number theory and technology.
- Student investigation on converting numbers in decimal and binary forms.
- Student investigation on arithmetic using numbers in binary form.
- Direct instruction on the utility of binary numbers.
- Direct instruction on basic number theory terms.
- Class discussion about examples of basic number theory terms.
- Guided instruction on the use and efficiency of the Sieve of Eratosthenes
- Student investigation and discovery into the number theory applications that lead to the solution to the "Fido Puzzle".
- Guided instruction on Fermat's Last Theorem through articles and the "Numberphile" video entitled "Simpsons and Fermat"
- Guided instruction on Graham's Number through articles and the "Numberphile" video entitled "Graham's Number".
- Direct instruction on the Fibonacci Sequence using the "Joy of Mathematics" video series as a guide.
- Direct instruction of properties of the Fibonacci Sequence using the book "Proofs That Really Count".
- Student discovery of strategies for optimal play in the Dominoes Game.
- Student discovery of how to win the Bridges Game.
- Student investigation on the Kaprekar process.
- Student investigation into the Tower of Hanoi problem and its connection to the binary number system through the article "The Icosian Game and the Tower of Hanoi".

Assessments:

Diagnostic:

- Teacher observation and questioning
- Unit 1 Common Assessment
- Pre Calculus Final Exam
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Formative:

- Teacher observation and questioning
- Homework assignments
- Graded homework assignments
- Group assignments
- Quizzes

Summative: Common assessment for Unit 2

Extensions:

- Extra problems from the textbook
- Teacher designed problems
- Various internet resources

Correctives:

- Re-teaching
- Teacher designed problems

Materials and Resources:

- Foundations of Higher Mathematics by Peter Fletcher and C. Wayne Patty, p. 22-26
- Hexaflexagons and Other Mathematical Diversions by Martin Gardner, p. 55-62
- Journey Through Genius – The Great Theorems of Mathematics by William Dunham, p.61-83
- "Joy of Mathematics" video series
- The Man Who Loved Only Numbers – The Story of Paul Erdös and the Search for Mathematical Truth by Paul Hoffman, p. 25-58
- The Mathematical Universe by William Dunham, p. 1-10, 193-200
- Penrose Tiles to Trapdoor Ciphers by Martin Gardner
- Proofs That Really Count by Arthur T. Benjamin

- Websites: http://illuminations.nctm.org/ActivityDetail.aspx?ID=40
  http://mathworld.wolfram.com/KaprekarRoutine.html
  http://www.digicc.com/fido/
  www.numberphile.com
  http://www.squiglysplayhouse.com/Games/Java/Bridges/Bridges.html

Unit 3: Methods of Proof

Time Range in Days: 15 Days

Overview: Students will use the principles of logic, proper mathematical notation, and various methods of proof to analyze, critique, and write proofs of given propositions.

Focus Questions:

- How are mathematical statements formally justified?
- What is an appropriate method for justifying a statement?
- What are some flaws in incorrect proofs?

Goals: Students will be able to

- Justify mathematical statements formally
- Determine an appropriate method for proving a proposition
- Critique and modify flawed proofs

Objectives: Students will be able to

- Explain the following methods of proof: Direct, Contrapositive, and Contradiction. (DOK Level Three)
- Use methods of proof to prove or disprove propositions. (DOK Level Four)
- Analyze and critique given proofs using the above methods. (DOK Level Four)
- Explain the proof that $\sqrt{2}$ is irrational. (DOK Level Three)
- Explain whether an irrational number raised to another irrational number can ever be a rational result. (DOK Level Three)
- Use the Element-Chasing method of proof to prove given propositions. (DOK Level Three)
- Explain how the proof of the 5 Platonic Solids is an example of an Element-Chasing proof. (DOK Level Three)
- Identify different polygons and polyhedra that are found within the structures of other polyhedra. (DOK Level Three)
- Explain and apply the concept of parity to solve problems. (DOK Level Three)
- Use parity to solve the Cop and Robber problem. (DOK Level Three)

Core Activities:

- Direct instruction on the direct, contrapositive, and contradiction methods of proof.
- Student investigation on the appropriateness of each method of proof for various propositions.
- Direct instruction on the irrationality of $\sqrt{2}$.
- Guided instruction on the Platonic Solids using the ”Numberphile” website.

- Student exploration and discovery on the different polygons and polyhedra that are found within the structures of other polyhedra.
- Student exploration and discovery of the concept of parity through the "Cop and Robber" Game.

Assessments:

Diagnostic:

- Teacher observation and questioning
- Unit 2 Common Assessment
- Pre Calculus Final Exam

Formative:

- Teacher observation and questioning
- Homework assignments
- Graded homework assignments
- Group assignments
- Quizzes

Summative: Common assessment for Unit 3

Extensions:

- Extra problems from the textbook
- Teacher designed problems

Correctives:

- Re-teaching
- Teacher designed problems

Materials and Resources:

- Foundations of Higher Mathematics by Peter Fletcher and C. Wayne Patty, p. 24-39
- The Mathematical Universe by William Dunham, p. 115-127
- Teacher designed worksheet on the structure of polyhedra
- Teacher designed Smartboard display of the "Cop and Robber Game"
- www.numberphile.com

Unit 4: Set Theory

Time Range in Days: 40 Days

Overview: Students will know basic principles of set theory, and use them to analyze and formulate propositions in set theory.

Focus Questions:

- How is a set defined?
- How are sets combined to create new sets?
- What general mathematical statements can be formulated about sets and families of sets?
- How can the Principle of Mathematical Induction be utilized to prove formulas for the set of natural numbers?
- How can theorems be justified using Proofs Without Words?
- How is the size of a set defined?
- How can the size of infinite sets be compared?
- Why do paradoxes arise from set theory?
- How are paradoxes resolved?

Goals: Students will be able to

- Define a set
- Apply various operations to create new sets
- Formulate general mathematics statements about sets
- Apply the Principle of Mathematical Induction
- Justify theorems using Proofs Without Words
- Analyze transfinite sets
- Explain and resolve paradoxes

Objectives: Students will be able to

- Define the following terms and use them appropriately to write and verbalize mathematical propositions: set, element, empty set, subset, proper subset, equal sets, and power set. (DOK Level Two)
- Analyze mathematical statements involving the terms above and formulate valid mathematical statements involving those terms. (DOK Level Three)
- Formulate valid mathematical statements involving the empty set. (DOK Level Three)
- Use standard mathematical notation for the empty set. (DOK Level One)
- Define universal set, and identify the universal set used for a given problem. (DOK Level One)
- Define intersection, union, and complement of a set. (DOK Level One)
- Draw a Venn Diagram for a given problem. (DOK Level Three)

- Interpret and explain a given Venn Diagram. (DOK Level Three)

- Use intersections, unions, and complements of sets to create more complex sets. (DOK Level Three)
- Define disjoint sets and use the term appropriately in mathematical statements, both verbal and written. (DOK Level One)
- Prove statements involving sets and operations on sets. (DOK Level Four)
- Analyze and critique given proofs involving sets. (DOK Level Four)
- Prove DeMorgan's Laws for sets. (DOK Level Four)
- Define families of sets and indexed families of sets. (DOK Level Two)
- Formulate unions and intersections of families of sets. (DOK Level Four)
- Prove theorems involving families of sets. (DOK Level Four)
- Analyze and critique given proofs involving families of sets. (DOK Level Four)
- Define an inductive set. (DOK Level Two)
- Explain the Principle of Mathematical Induction. (DOK Level Three)
- Use the Principle of Mathematical Induction to prove conjectures. (DOK Level Four)
- Explain the Extended Principle of Mathematical Induction. (DOK Level Three)
- Use the Extended Principle of Mathematical Induction to prove conjectures. (DOK Level Four)
- Analyze and critique given proofs that use the Principle of Mathematical Induction. (DOK Level Four)
- Prove algebraic inequalities that are true for given values of the variables. (DOK Level Three)
- Interpret "Proofs Without Words" for theorems in geometry, algebra, and number theory. (DOK Level Four)
- Create "Proofs Without Words" to justify mathematical conjectures. (DOK Level Four)
- Analyze and explain "Proofs Without Words" for the Pythagorean Theorem. (DOK Level Four)
- Explain the difficulties mathematicians had in defining the limit of a function. (DOK Level Three)
- Analyze Karl Weierstrass' formal definition of a limit of a function. (DOK Level Three)
- Define equinumerous sets in terms of the concept of one-to-one. (DOK Level Two)
- Explain the objections posed by mathematicians to the work of Georg Cantor. (DOK Level Two)
- Analyze and explain the proofs that the following sets are countable: the even natural numbers, the integers, and the rational numbers. (DOK Level Four)
- Explain the concept of countably infinite. (DOK Level Three)

- Analyze and explain the proofs that the following sets are uncountable: any interval of real numbers, the real numbers, and the irrational numbers. (DOK Level Four)
- Explain the difference between algebraic numbers and transcendental numbers and their relevance to the work of Cantor. (DOK Level Three)
- Analyze and explain the proof that the set of numbers in a given interval is equinumerous to the set of ordered pairs in a given square. (DOK Level Four)
- Prove that the power set of any set is larger than the set itself. (DOK Level Four)
- Apply the above principle to conclude that there is an infinite number of different sized infinite sets. (DOK Level Four)
- Explain the phrase "non-denumerability of the continuum". (DOK Level Three)
- Know the history of Cantor and his development of the theory of infinite sets. (DOK Level Two)
- Define paradox and give examples of paradoxes. (DOK Level Three)

Core Activities:

- Direct instruction on the definitions of set, element, subset, proper subset, equal sets, power set, union, intersection, and complement.
- Student investigation and discussion about proper set theory notation.
- Guided investigation on general set theory statements and their relationship to similar statements in logic, including DeMorgan's Laws.
- Guided practice in developing Venn Diagrams to represent unions, intersections, and complements of sets.
- Direct instruction on proving propositions in set theory.
- Class discussion on critiquing flawed proofs in set theory.
- Direct instruction on indexed families of sets and their unions and intersections.
- Direct instruction on the Principle of Mathematical Induction, using the "Domino Effect" as an analogy.
- Guided practice in proving formulas for the natural numbers using the Principle of Mathematical Induction.
- Use the Points on a Circle/Number of Regions problem to establish that conjectures based on a few examples must be made cautiously and verified mathematically before accepting as true.
- Class discussion on Proofs Without Words to justify mathematical statements.
- Group activity in which students discuss and discuss Proofs Without Words for the Pythagorean Theorem.

- Direct instruction on the historical difficulties mathematician had in formally defining a limit, using the book "Journey Through Genius" as a guide.
- Class investigation and discussion about matching up certain sets (naturals, whole numbers, integers, and rational numbers) in a one to one correspondence.
- Direct instruction and class discussion on the lack of a one-to-one correspondence between the real numbers and the natural numbers.
- Direct instruction and class discussion on the idea that the size of the power set of a set is always greater than the size of the set itself.
- Student investigation and class discussion of examples of paradoxes, including Zeno's paradox, Protagoras' paradox, the Pop Quiz Paradox, and Simpson's Paradox.
- Direct instruction and class discussion of the various forms of Russell's Paradox and how they apply to problems in the theory of infinite sets.

Assessments:

Diagnostic:

- Teacher observation and questioning
- Unit 3 Common Assessment
- Pre Calculus Final Exam

Formative:

- Teacher observation and questioning
- Homework assignments
- Graded homework assignments
- Group assignments
- Quizzes

Summative: Common assessment for Unit 4

Extensions:

- Extra problems from the textbook
- Extra problems from the book "Proofs Without Words"
- Teacher designed problems

Correctives:

- Re-teaching
- Teacher designed problems

Materials and Resources:

- Foundations of Higher Mathematics by Peter Fletcher and C. Wayne Patty, p. 41-59
- The Mathematical Universe by William Dunham, p. 213-223, 261-262
- Selected problems from Proofs Without Words by Roger B. Nelsen
- Journey Through Genius – The Great Theorems of Mathematics by William Dunham, p. 245-283
- History of Mathematics by David M. Burton, p. 589-628
- "Paradox Lost" – by Ian Stewart
- Various websites demonstrating Simpson's Paradox

Unit 5: Flatland

Time Range in Days: 5 Days

Overview: Students will read Flatland by Edwin Abbott, and discuss the idea of higher dimensions, as well as the geometry and life in Flatland.

Focus Questions:
- What is life like in Flatland?
- Why is the social structure in Flatland the way it is?
- How did the square become aware of another dimension?
- What analogies and examples are given that describe higher dimensions?
- What implications does the book suggest to our three dimensional world?

Goals: Students will be able to
- Explain the social structure of Flatland. (DOK Level Three)
- Analyze the issues that arise from life in Flatland. (DOK Level Three)
- Explain the social commentary provided by Edwin Abbott. (DOK Level Three)
- Describe the events that led to the square questioning a higher dimension. (DOK Level Three)
- Explain how the square came to understand a higher dimension. (DOK Level Three)
- Describe the interactions the square had with the inhabitants of Pointland and Lineland. (DOK Level Three)
- Analyze the implications of higher dimensions to our world. (DOK Level Four)

Core Activities:
- Individual reading of Flatland by Edwin Abbott.
- Class discussion on the life, history, and social structure of inhabitants of Flatland and analysis of the social commentary presented in Flatland.
- Class discussion of the dimensional perspectives presented in Flatland.
- Class discussion of the article "Welcome to the Dark Side" by Ron Rosenbaum

Assessments:

Diagnostic:
- Teacher observation and questioning

Formative:
- Teacher observation and questioning

- Homework assignments
- Class discussions

Summative: Common assessment for Unit 5

Extensions:

- Videos on the www.numberphile.com website

Correctives:

- Re-teaching
- Teacher designed problems

Materials and Resources:

- Flatland by Edwin Abbott
- “Welcome to the Dark Side” by Ron Rosenbaum
- Videos on the www.numberphile.com website